

Phase Transition in Reissner-Nordstrom-AdS Black Hole With Quintessence Matter

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ABSTRACT

We study the second order phase transition of the Reissner-Nordstrom-AdS black hole in quintessence matter. Using the first law of thermodynamics, some expressions of thermodynamics in this black hole are obtained. We study the phase transition condition under different parameter ω or other parameters. For three special cases, we obtain the conditions of phase transition and analysis in detail. We show the relations of the temperature T , heat capacity C_Q, C_ϕ with the entropy of black hole by figures. We also present the relation of phase transition point S with α, Q, β and ω .

Subject headings: Thermodynamics, Reissner-Nordstrom-AdS black hole, Quintessence matter, Phase transition, condition

1. INTRODUCTION

Black hole thermodynamics is one of the important topic in modern physics and was widely studied in many years. The laws of black hole dynamics and thermodynamics were analyzed by Bekenstein and Hawking ([Bekenstein \(1972, 1974, 1973\)](#); [Hawking \(1971, 1974\)](#)). The four laws of black hole thermodynamics have been discussed ([Bardeen et al. \(1973\)](#)). These results were already generalized to other black holes. The black holes have also the phase transition, for example, Reissner-Nordstrom black hole, Kerr black hole and Kerr-Newman black hole ([Caldarelli et al. \(2000\)](#); [Davies \(1989, 1978, 1977\)](#)). These works were also generalized to other black holes or general situation ([Hawking & Page \(1982\)](#); [Hut \(1977\)](#); [Liao et al. \(2016\)](#); [Mandal et al. \(2016\)](#)).

From the recent observation, we know that our universe is dominated by Dark Energy ([Komatsu et al. \(2011\)](#); [Riess et al. \(2004\)](#)). The dark energy makes the universe to be the

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accelerated expansion. The state of equation of dark energy is very close to cosmological constant or vacuum energy (Peebles & Ratra (2003)). But the dynamics of dark energy is more possible (Copeland et al. (2006)), such as quintessence (Copeland et al. (2006)). The dark energy with quintessence could affect the black hole spacetime (Stuchlík (2005)). For the schwarzschild black hole in quintessence field, the modified black hole metric have been obtained by Kiselev (Kiselev (2003)). Recently the rotational quintessence black hole and Kerr-Newman black hole solution are also obtained (Toshmatov et al. (2015); Xu & Wang (2016)). For the schwarzschild black hole surrounded by quintessence matter, the thermodynamics and phase transition have been discussed (Tharanath et al. (2014); Ghaderi & Malakolkalami (2016,?)). The Rerssner-Nordstrom black hole and Rerssner-Nordstrom black hole situation have been studied (Ghaderi & Malakolkalami (2016); Wei & Chu (2011); Wei & Ren (2013); Thomas et al. (2012)), but they only correspond to very special case with $\omega = -2/3$. For Reissner-Nordstrom-AdS black holes, the thermodynamics have been investigated (Pradhan (2016)). In this paper we study the phase transition for this black hole.

In this paper, we study the thermodynamics and phase transition of Reissner-Nordstrom-AdS black hole in quintessence matter. In section 2, we introduce the Reissner-Nordstrom-AdS black hole with quintessence matter. In section 3, we calculate some thermodynamical quantities, including Hawking temperature, potential and heat capacity for the parameters Q, α, β and ω . For three special situations, we discuss the condition of phase transition. The summary is in last section.

2. THERMODYNAMICS IN QUINTESSENCE KERR-NEWMAN BLACK HOLE

The static spherically symmetric solution of Einstein equation for a Reissner-Nordstrom-AdS black hole in quintessence matters have been obtained by Kiselev (Kiselev (2003)) as

$$ds^2 = f(r)dt^2 - f^{-1}(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \beta r^2 - \alpha r^{-3\omega-1} \quad \beta = \frac{1}{l^2}, \quad (2)$$

M and Q are the black hole mass and charge. $\beta = \frac{1}{l^2}$ is related to the cosmological constant. ω is the state parameter for quintessence matter under the state equation $p = \omega\rho$. If ω satisfies $-1 < \omega < -1/3$, the quintessence will make the universe acceleration. The parameter α is related to the energy density of quintessence matter.

The horizon of this black hole satisfies the following equation

$$1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \beta r^2 - \alpha r^{-3\omega-1} = 0. \quad (3)$$

Usually this equation has three roots corresponding to inner, event and cosmological horizons if $-1 < \omega < -1/3$, but we pay attention to inner and event horizons.

The entropy of the Reissner-Nordstrom-AdS black hole satisfies the area law in thermodynamics given by

$$A = 4S = 4\pi r_+^2. \quad (4)$$

Here we only consider event horizon, while Cauchy and cosmological horizons will satisfy the same law.

3. PHASE TRANSITION

From the equation (3) and (4), the mass of black hole with the entropy is given by

$$\begin{aligned} M &= \frac{r_+}{2} \left(1 + \frac{Q^2}{r_+^2} + \beta r_+^2 - \alpha r_+^{-3\omega-1} \right) \\ &= \frac{1}{2} \left[\sqrt{\frac{S}{\pi}} + Q^2 \sqrt{\frac{\pi}{S}} + \beta \left(\frac{S}{\pi} \right)^{3/2} - \alpha \left(\frac{S}{\pi} \right)^{-3\omega/2} \right], \end{aligned} \quad (5)$$

which is called as the generalized Smarr mass formula (Smarr (1973,?)) for Reissner-Nordstrom-AdS black hole. From the first law of black hole thermodynamics with

$$dM = TdS + \phi dQ, \quad (6)$$

we can define the following thermodynamical quantities as:

Energy density

$$\rho = -\frac{\alpha}{2} \frac{3\omega}{r^{3(\omega+1)}} = -\frac{3\alpha\omega}{2} \left(\frac{S}{\pi} \right)^{\frac{3\omega+3}{2}}. \quad (7)$$

Semi-Hawking temperature

$$T = \left(\frac{\partial M}{\partial S} \right)_Q = \frac{1}{4(\pi S)^{3/2}} \left[\pi S - \pi^2 Q^2 + 3\beta S^2 + 3\alpha\omega\pi \frac{3\omega+3}{2} \frac{1-3\omega}{S^2} \right]. \quad (8)$$

Potential

$$\phi = \left(\frac{\partial M}{\partial Q} \right)_S = Q \sqrt{\frac{\pi}{S}}. \quad (9)$$

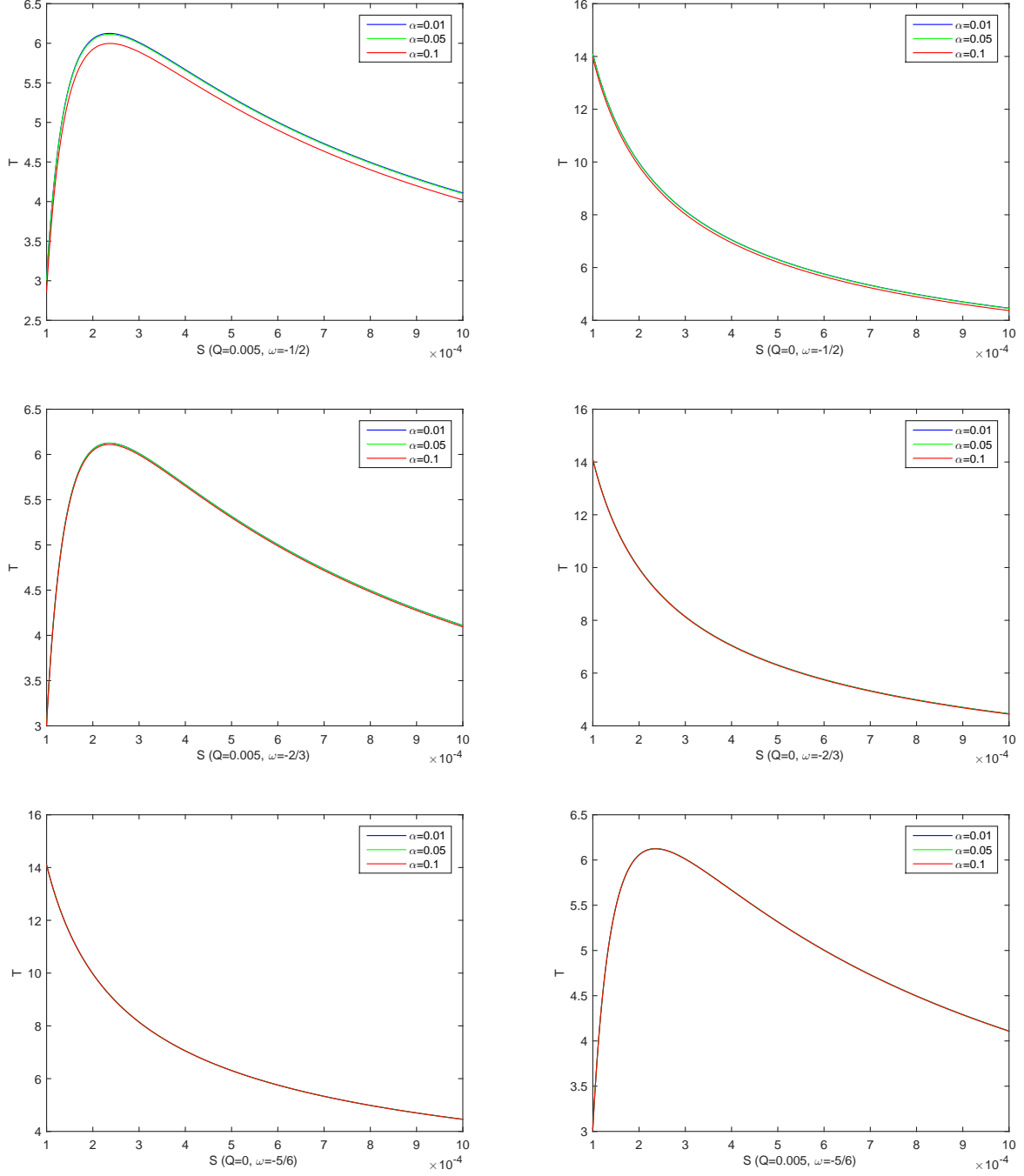


Fig. 1.— Hawking temperature T versus black hole entropy S for different parameters Q and ω ($\alpha = 0.1$).

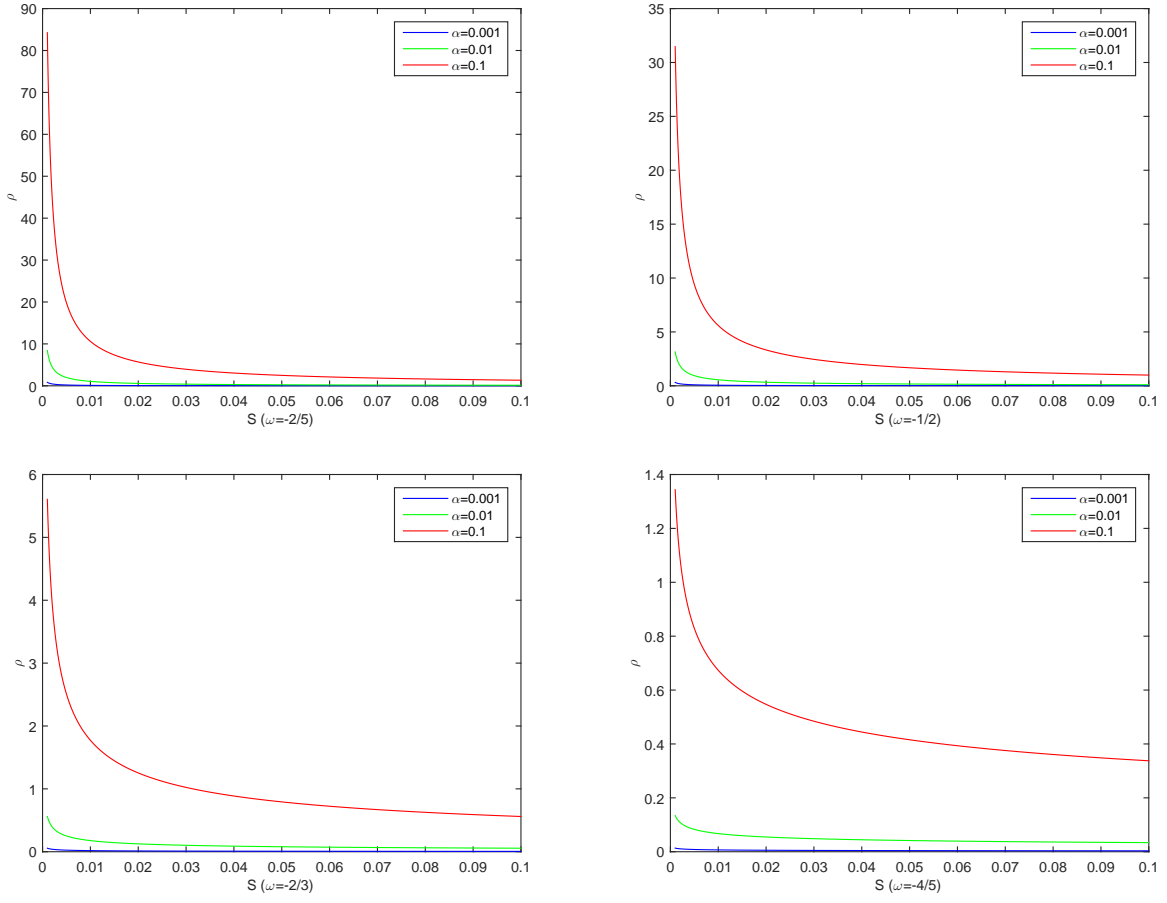


Fig. 2.— Energy density ρ versus entropy S for different parameters α and ω .

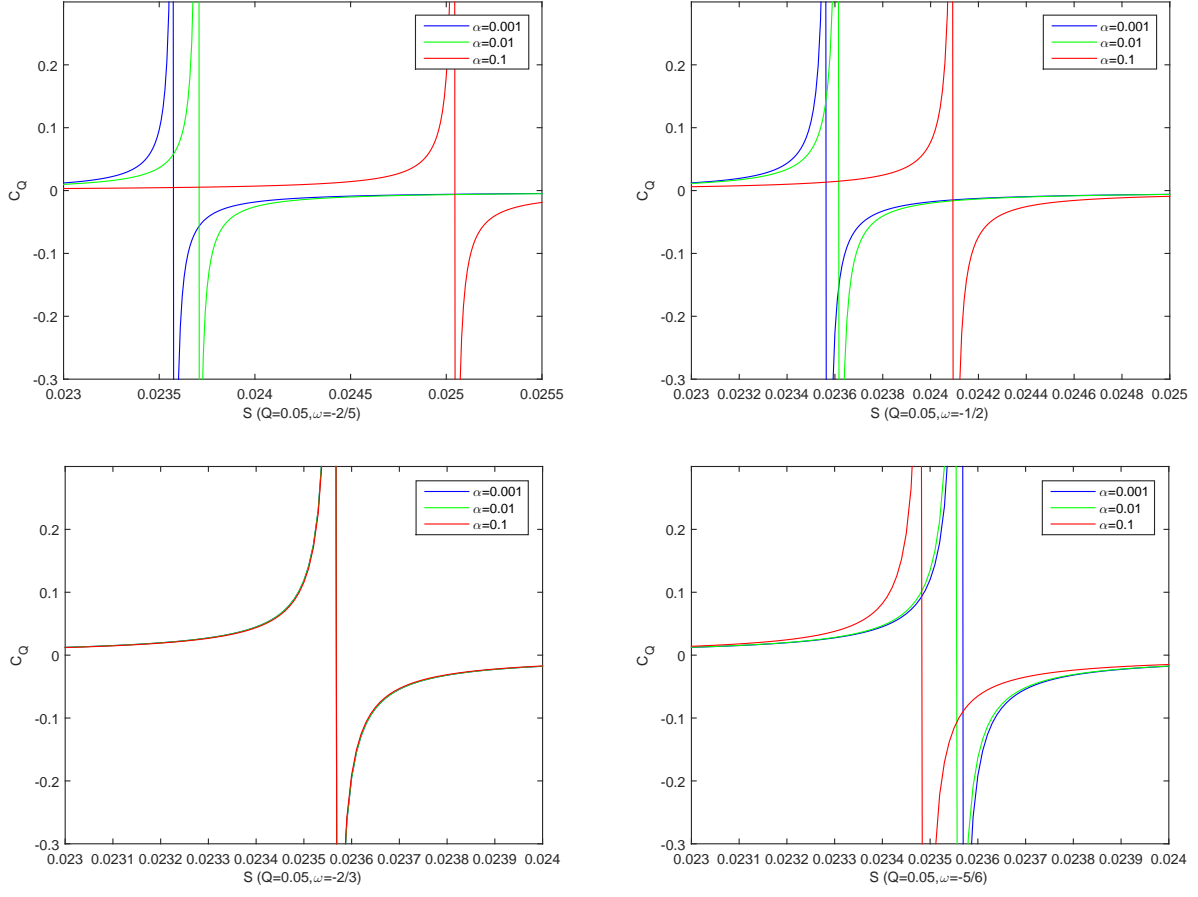


Fig. 3.— Heat capacity C_Q versus entropy S for different parameters α and ω ($Q = 0.05$).

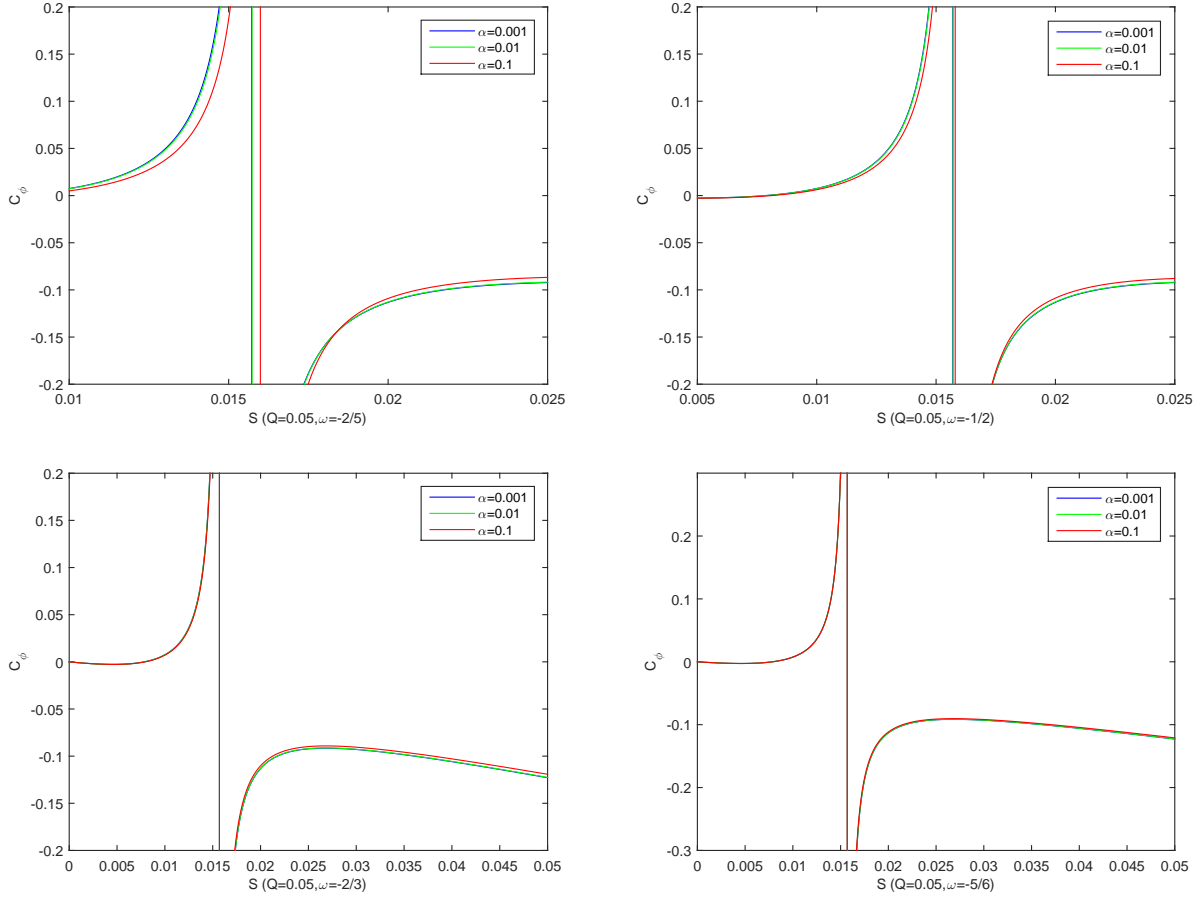


Fig. 4.— Heat capacity C_ϕ versus entropy S for different parameters α and ω ($Q = 0.05$).

When Q is constant, the heat capacity is given by

$$C_Q = T\left(\frac{\partial S}{\partial T}\right)_Q = -2S \frac{S - \pi Q^2 + \frac{3\beta S^2}{\pi} + 3\alpha\omega\pi \frac{3\omega+1}{2} \frac{1-3\omega}{S} \frac{1-3\omega}{2}}{S - 3\pi Q^2 - \frac{3\beta S^2}{\pi} + 3\alpha\omega(2+3\omega)\pi \frac{3\omega+1}{2} \frac{1-3\omega}{S} \frac{1-3\omega}{2}}. \quad (10)$$

When ϕ is constant, the heat capacity is written as

$$C_\phi = T\left(\frac{\partial S}{\partial T}\right)_\phi = 2S \frac{S - \pi Q^2 + \frac{3\beta S^2}{\pi} + 3\alpha\omega\pi\left(\frac{\pi}{S}\right) \frac{3\omega-1}{2}}{-S + 2\pi Q^2 + \frac{3\beta S^2}{\pi} - 3\alpha\omega(2+3\omega)\left(\frac{\pi}{S}\right) \frac{3\omega-1}{2}}. \quad (11)$$

The Semi-Hawking temperature T versus black hole entropy S for different parameters α, Q and ω is shown in Figure 1. It is shown that when the state parameter is close to cosmological constant, the Hawking temperature will change small, which is the same with the paper (Thomas et al. (2012)). The energy density versus the entropy for different parameters is shown in Figure 2. When Q and ϕ are constants, the heat capacities for different parameters are shown in Figure 3 and 4. Because the cosmological constant is very small, their changes are also small.

For phase transition, the charge Q is constant. When the entropy satisfies the following equation

$$S - 3\pi Q^2 - \frac{3\beta S^2}{\pi} + 3\alpha\omega(2+3\omega)\pi \frac{3\omega+1}{2} \frac{1-3\omega}{S} \frac{1-3\omega}{2} = 0, \quad (12)$$

the phase transition will happen and depend on parameters Q, β, α and ω (Pradhan (2016)). In Figure 5, we plot the critical point S versus the charge Q . We find that the critical point increases with charge Q . In Figure 6, we plot the critical point S versus parameter α when other parameters are fixed. When ω is close to $-1/3$, the critical point increases with α . But when ω passes $-2/3$ and is close to -1 , the critical point decreases with α . Because the cosmological constant is small, the critical point changes less. Through numerical calculation, we find that there are two critical points. From Figure 8, we find that the critical point changes less when ω is close to -1 , but it changes very largely when ω is close to $-1/3$.

In order to know the critical point in analysis, we use three examples to show these property.

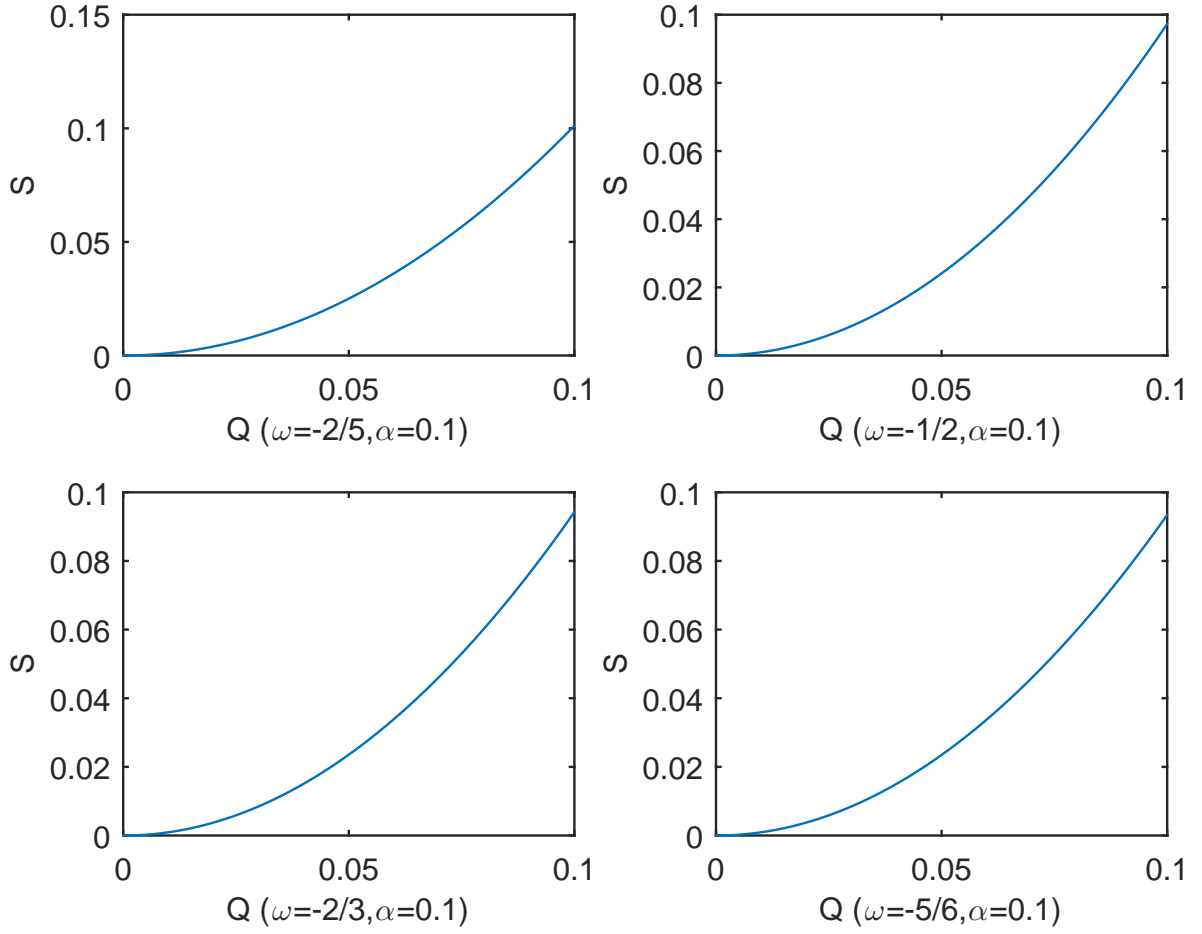


Fig. 5.— Critical point S versus charge Q for different parameters α and ω (when fixed β).

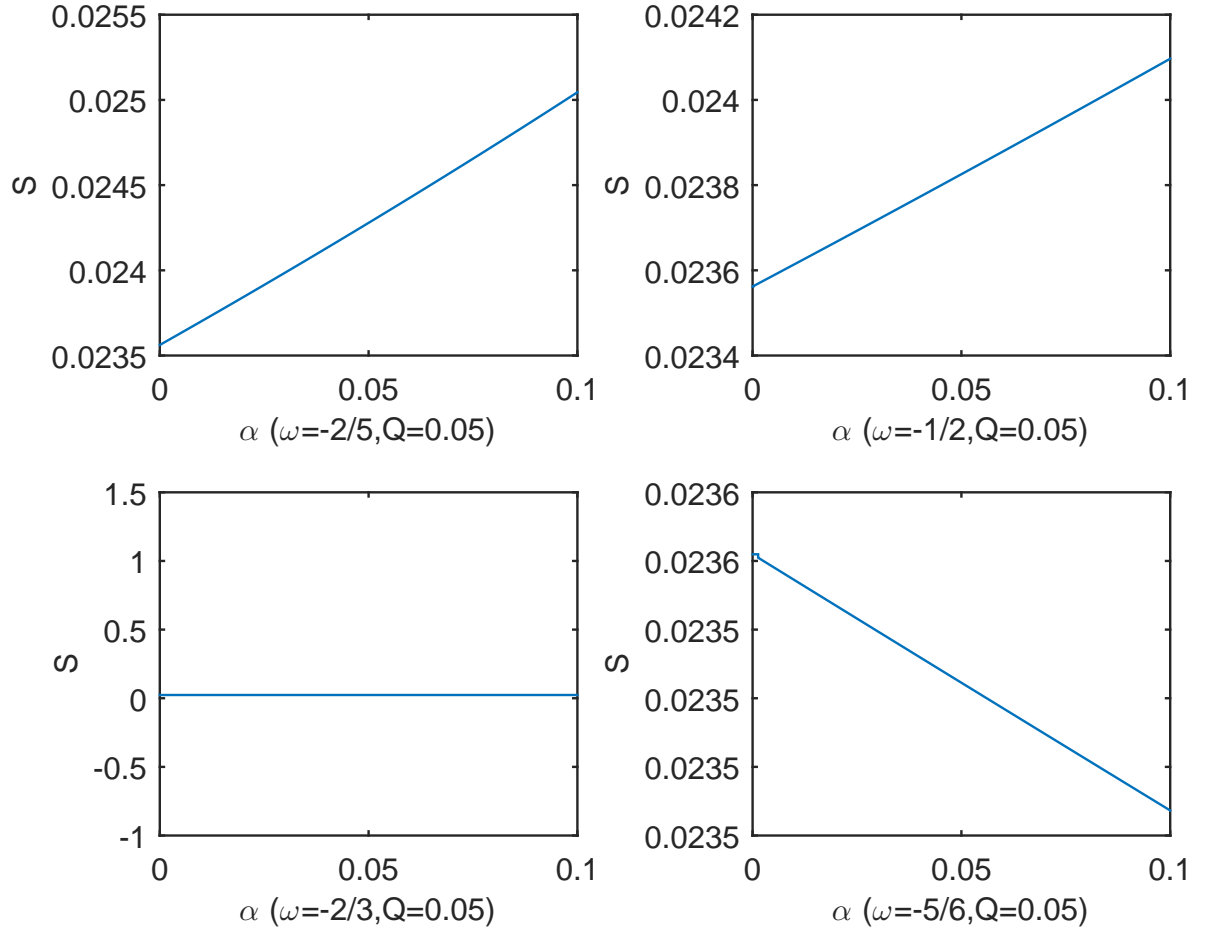


Fig. 6.— Critical point S versus parameter α for different parameters ω (when fixed Q, β).

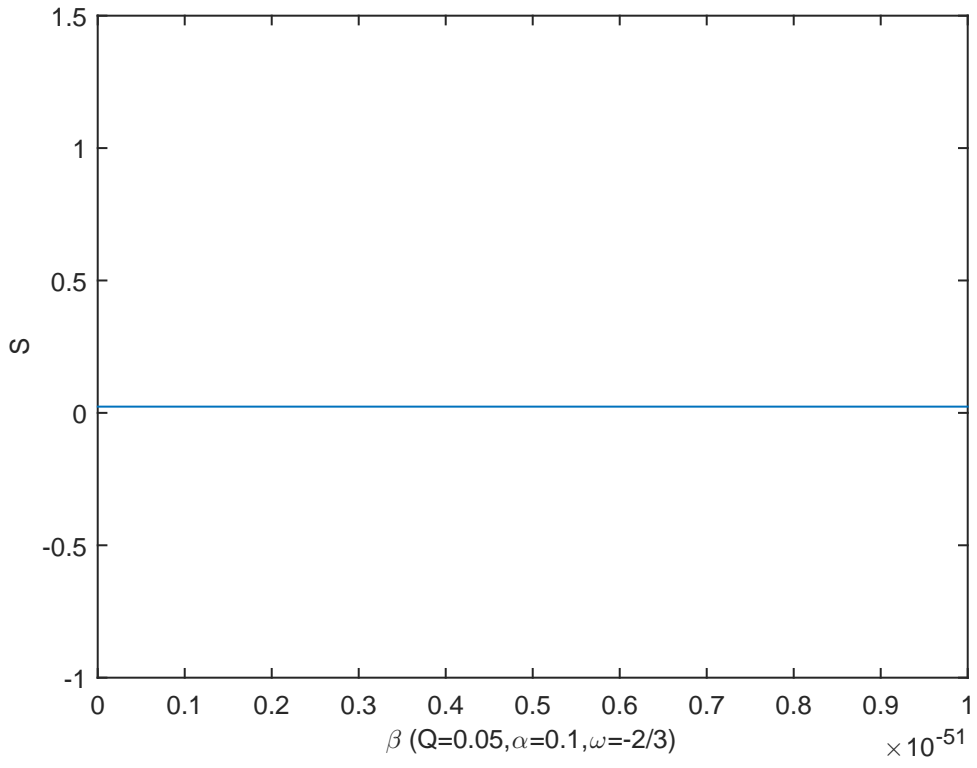


Fig. 7.— Critical point S versus parameter β . Because the value of β is small, the influence is also very small.

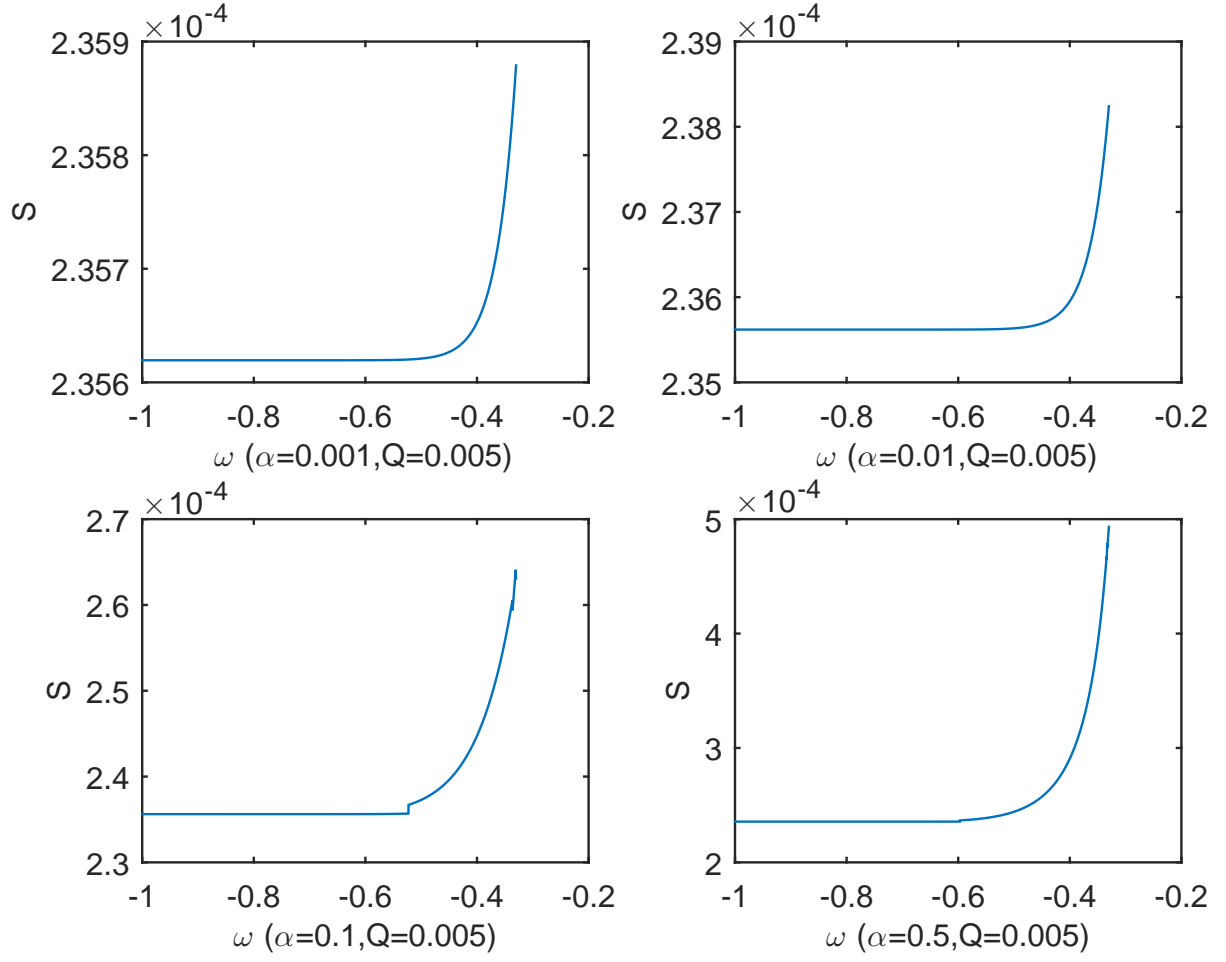


Fig. 8.— Critical point S versus state parameter ω for different parameters α . Other parameters β, Q are fixed.

Case one: $\omega = -1/3$. This case is the special limit for quintessence around Reissner-Nordstrom-AdS black hole. If the equation (12) exists root, two roots are given by

$$\begin{aligned} S_+ &= \frac{\pi}{6\beta}(1 - \alpha + \sqrt{(\alpha - 1)^2 - 36\beta Q^2}), \\ S_- &= \frac{\pi}{6\beta}(1 - \alpha - \sqrt{(\alpha - 1)^2 - 36\beta Q^2}), \end{aligned} \quad (13)$$

and the phase transition condition is given by

$$\alpha - 1 \geq 6Q\sqrt{\beta} \quad \text{and} \quad \beta \neq 0. \quad (14)$$

For the quintessence around Reissner-Nordstrom-dS black hole, β become $-\beta$. We then obtain

$$\begin{aligned} S &= \frac{\pi}{6\beta}(\sqrt{(\alpha - 1)^2 - 36\beta Q^2} - 1 + \alpha), \\ (\alpha - 1)^2 + 36\beta Q^2 &> 0. \end{aligned} \quad (15)$$

From above analysis, we can know that the critical point becomes one from RN-AdSQ to RN-dSQ. The cosmological constant will affect the phase transition largely. If $\beta = 0$ the critical point will reduce to that of RNQ case, and the root equals to $S = 3\pi Q^2/(1 - \alpha)$.

Case two: $\omega = -1$. This case is close to the cosmological constant. Two roots are given by

$$S = \frac{\pi}{6(\alpha - \beta)}(\sqrt{1 + 36Q^2(\alpha - \beta)} - 1), \quad (16)$$

and the phase transition condition becomes

$$36Q^2(\beta - \alpha) \leq 1. \quad (17)$$

For dSQ case, we change β to $-\beta$. The phase transition condition is then given by

$$S = \frac{\pi}{6(\alpha + \beta)}(\sqrt{1 + 36Q^2(\alpha + \beta)} - 1) \quad (18)$$

Case three: $\omega = -2/3$. For this case, two roots are

$$\begin{aligned} S_+ &= \frac{\pi}{6\beta}(1 + \sqrt{1 - 36\beta Q^2}), \\ S_- &= \frac{\pi}{6\beta}(1 - \sqrt{1 - 36\beta Q^2}), \end{aligned} \quad (19)$$

and the condition of phase transition becomes

$$36Q^2\beta \leq 1 \quad \text{and} \quad \beta \neq 0. \quad (20)$$

For dSQ case, we change β to $-\beta$. The phase transition condition is written as

$$S = \frac{\pi}{6\beta}(\sqrt{1 + 36\beta Q^2} - 1) \quad (21)$$

For $\omega = -2/3$, the parameter α will vanish and correspond to RN-AdS situation. We find $\omega = -2/3$ to be the critical case under quintessence situation $-1 < \omega < -1/3$.

When ϕ is constant, we consider how does C_ϕ diverge. The critical points satisfy the equation

$$-S + 2\pi Q^2 + \frac{3\beta S^2}{\pi} - 3\alpha\omega(2 + 3\omega)\left(\frac{\pi}{S}\right)\frac{3\omega - 1}{2} = 0, \quad (22)$$

which is very similar to the equation (12).

If the quintessence vanishes, the black hole will reduce to RN-AdS black hole (Pradhan (2016)). If the cosmological constant ignores, it reduce to RN in quintessence (Thomas et al. (2012)). Because the cosmological constant is small, the difference between RN-AdS and RN in quintessence is also small. The condition of phase transition is always satisfied. But if the cosmological constant is considered, two critical points will occur and the phase diagram will be changed.

4. SUMMARY

In this paper, we have studied the phase transition in Reissner-Nordstrom-AdS black hole in quintessence matters. From the first law of thermodynamics, we calculate some thermodynamical quantity including Semi-Hawking temperature, potential and heat capacity (two situation) as a function of entropy. Through studying the heat capacity, The condition of phase transition for different parameter ω also obtain. We also show the relation of temperature versus black hole entropy, heat capacity versus black hole entropy, critical point versus parameters α, β, Q and ω . The critical points shift largely with Q and α , but the critical point shifts small with ω and β . When ω is close to $-1/3$, the critical point increases with α . When ω passes $-2/3$ and is close to -1 , the critical point decreases with α .

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